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Thermofield dynamics in the theory of magnetic polaron mobility in ferromagnetic semiconductors

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Abstract. We used the method of thermofield dynamics to calculate the magnetic polaron mobility in a ferromagnetic semiconductor at temperatures much lower than the Curie temperature. We obtained an exact equation for mobility in the lowest order of electron occupation numbers. Other parameters were assumed to be arbitrary quantities. The results showed that the gap in the magnon spectrum changes the low-temperature mobility asymptotics from polynomial to exponential form.

1. Introduction

Over the last 30 years, many people have studied the magnetic polaron problem, in which the conduction electron mobility in a magnetic semiconductor is affected by the lattice magnetization created by the electron itself. The magnetic polaron problem has all the difficulties inherent in the particle–boson interactions involving strong coupling, so it would seem that we cannot construct a rigorous theory and must instead be satisfied with various special limiting cases and variational methods.

Fortunately, because of spin conservation, ferromagnetic semiconductors are an exception to this rule and the Schrödinger equation for magnetic polaron states can be solved rigorously for any spin value of the spin–electron coupling at $T = 0$ [1, 2] (see also [3–5]). At zero temperature, magnetic polarons are undamped quasiparticles with an effective mass somewhat larger than that of a free electron, and the bottom of the magnetic polaron band is somewhat lower.

In addition to the renormalization of both the mass and the ground state level at finite temperatures, polarons have a finite lifetime and magnon contribution to their mobility. Increasing temperature causes a sharp drop in the polaron lifetime, and at some temperatures it may be of the same order as the magnetic polaron energy. The spectrum ceases its free-quasiparticle behaviour and most of the carriers are trapped by magnetization fluctuations [6–7].

The magnetic polaron problem has been treated in various ways. Most papers have dealt with extreme cases of either weak spin–electron coupling (wide electron bands) [8–10] or very strong spin–electron coupling (narrow bands) [11, 12]. For the latter case, a low value of $1/2S$ was used as the perturbation parameter (S is the atomic spin). Regardless of the fact that the s–f exchange integral might be large, the effective mass renormalization is small in the limit $1/2S \rightarrow 0$.

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In a number of papers [13–15], the relation between the band width and the s–f exchange constant A is assumed to be arbitrary, but the approximations used give rise to uncontrolled errors in the final results. The authors of [16] calculated the spectrum and damping of a magnetic polaron at low temperatures without using any uncontrolled approximations. The authors of [17] claimed to do the same, but the results of these two papers are not in agreement.

The authors of [16] used expansion of path integrals over magnon numbers, but they assumed that the magnon energies would be negligible ($\omega_q = 0$).

Previous papers that dealt with the intermediate coupling case were concerned only with the one-particle Green function, and even so, no attempts were made to calculate polaron mobility.

The aim here is to fill this gap in our knowledge (at least for low temperatures) by calculating the spin polaron damping and its mobility while explicitly taking into account magnon energies. In the limit $\omega_q = 0$ our results are in agreement with those of [16]. The only restriction on the values of parameters is that the temperature must be low. All other relations between the parameters of the Hamiltonian are assumed to be arbitrary.

The technique of thermofield dynamics is useful for obtaining low-temperature expansions in cases where the zero-temperature problem has an exact solution [18] (e.g. the case of the magnetic polaron problem in ferromagnets).

The thermofield dynamics method is based on the equivalence of the calculation of traces of statistical matrices and the calculation of the matrix elements of some vacuum state dependent on temperature (for details see [18]). Therefore, it is possible to introduce a temperature-dependent 'Hamiltonian'. This 'Hamiltonian' coincides with a true Hamiltonian in the limit $T = 0$. Using the standard thermofield dynamics procedure, we introduce a Hamiltonian dependent on magnon number n. We consider the terms proportional to n that are small at low temperatures as perturbative. We treat the terms of zeroth order in n as the unperturbed part of the Hamiltonian. We diagonalize the unperturbed Hamiltonian by the appropriate canonical transformation and we then develop a perturbation technique with n as the only small parameter. To the first non-vanishing order, we find the magnetic polaron damping and mobility.

As a rule, magnon energies are small compared with electron energies, but we show that magnon energies must be taken into account to obtain the low-temperature asymptotics of polaron damping. If we regard ω_q as negligibly small, then the spin rotation invariance of the s–f exchange Hamiltonian causes cancellations in the equations for polaron damping. The amplitude of the electron–magnon scattering vanishes when wavevectors tend towards zero. Therefore, in the equations for effective mass and damping, the temperature correction terms have high powers (e.g. the damping $\gamma \propto T^{7/2}$).

However, if we take into account the magnetic anisotropy energy, the rotation invariance breaks down. The magnon spectrum has a gap of type $\omega_q = \omega_0 + \alpha q^2$, $\omega_0 \neq 0$ and the low-temperature asymptotic must be of another type. We show that temperature correction terms of the order $\omega_0 T^{3/2} \exp(-\omega_0/kT)$ survive in the expression for damping.

2. Thermofield dynamics transformations

The Hamiltonian for a conduction electron interacting with the spins of magnetic atoms is given by

$$H = \sum_{p\sigma} E(p) a_{p\sigma}^{+} a_{p\sigma} + A(N)^{-1/2} \sum_{p\sigma} (a_{k\mathrm{u}}^{+} S_{p-k}^{-} a_{p\mathrm{d}} + a_{p\mathrm{d}}^{+} S_{p-k}^{+} a_{k\mathrm{u}})$$
$$+ A(N)^{-1/2} \sum_{pk} (a_{k\mathrm{u}}^{+} S_{k-p}^{z} a_{p\mathrm{u}} - a_{k\mathrm{d}}^{+} S_{k-p}^{z} a_{p\mathrm{d}}) + H_{\mathrm{H}}. \qquad (1)$$

Here $a_{p\sigma}^{+}$ and $a_{p\sigma}$ (σ = up (u) or down (d)) are conduction electron operators with spin σ and wavevector p; $A > 0$ is the s–f exchange integral, N is the number of magnetic atoms in the crystal; S^{+}, S^{-} and S^{z} are the Fourier images of the lattice spin operators; and H_{H} is the Hamiltonian of the atomic spins. For convenience, we transform S^{+}, S^{-} and S^{z} to Bose operators using the Dyson–Maleev transformation, in the site representation

$$S_n^{-} = (2S)^{1/2} c_n^{+} (1 - c_n^{+} c_n / 2S) \qquad S_n^{+} = (2S)^{1/2} c_n \qquad S_n^{z} = S - c_n^{+} c_n. \qquad (2)$$

After the Fourier transformation $c_k^{+} = (N)^{-1/2} \Sigma\, c_n^{+} \exp(\mathrm{i} k r_n)$ the Hamiltonian (1) takes the form

$$H = \sum E(p\sigma) a_{p\sigma}^{+} a_{p\sigma} + \frac{(2S)^{1/2} A}{N} \sum (a_{k\mathrm{u}}^{+} c_{p-k}^{+} a_{p\mathrm{d}} + a_{p\mathrm{d}}^{+} c_{p-k} a_{k\mathrm{u}})$$
$$+ \frac{A}{N} \sum_{kp} (a_{k\mathrm{d}}^{+} a_{p\mathrm{d}} - a_{k\mathrm{u}}^{+} a_{p\mathrm{u}}) c_q^{+} c_{k-p+q} - \frac{A}{N(2SN)^{1/2}}$$
$$\times \sum_{k k_1 q q_1} a_{k_1\mathrm{u}}^{+} c_{q_1}^{+} c_{k-k_1+q-q_1}^{+} c_q a_{k\mathrm{d}} + H_{\mathrm{H}}. \qquad (3)$$

In the above equation we have $H_{\mathrm{H}} = \Sigma\, \omega(q) c_q^{+} c_q$, and $E(p\sigma) = E(p) + AS\delta_\sigma$, in which $\delta_{\mathrm{u}} = 1$ and $\delta_{\mathrm{d}} = -1$.

In (3) H_{H} is the Hamiltonian for free magnons.

To introduce thermofield dynamics transformations, we take this Hamiltonian (3) and add the tilded Hamiltonian [18]

$$\tilde{H}_{\mathrm{H}} = -\sum \omega(q) \tilde{c}_q^{+} \tilde{c}_q \qquad (4)$$

that contains a Bose field with negative energies. Adding $\tilde{H}_{\mathrm{H}}$ to (3) does not affect its eigenstates since it does not contribute to the interaction.

We wish to solve the eigenvalue problem for the Hamiltonian (3) on a single-fermion subspace, so we do not need to use tilded electron operators.

To connect the variables of (4) with those of (3), the Bogolubov transformation must be done in the following way:

$$c_q = \cosh\theta(q)\, \tilde{c}_q(\beta) + \sinh\theta(q)\, \tilde{c}_q^{+}(\beta) \qquad (5a)$$

$$\tilde{c}_q = \cosh\theta(q)\, \tilde{c}_q(\beta) + \sinh\theta(q)\, \tilde{c}_q^{+}(\beta) \qquad (5b)$$

$$\left.\begin{aligned} &\sinh\theta(q) = n_q^{1/2} = (\mathrm{e}^{\beta\omega(q)} - 1)^{-1/2} \qquad \beta = 1/kT \\ &\cosh\theta(q) = (1 + n_q)^{1/2}. \end{aligned}\right. \qquad (5c)$$

Following [18], the temperature-dependent Bose operators $\tilde{c}_q(\beta)$, $\tilde{c}_q^{+}(\beta)$ and $\tilde{c}_q^{+}(\beta)$, $c_q(\beta)$ are, respectively, the hole and magnon thermofield operators. The vacuum state $|\Phi\rangle$ for such operators (i.e. $c_q(\beta)|\Phi\rangle = 0$ and $\tilde{c}_q(\beta)|\Phi\rangle = 0$) depends on the temperature and coincides with the thermodynamic equilibrium state, so that the magnon numbers are given in the usual way as $\langle\Phi| c_q^{+} c_q |\Phi\rangle = n_q$. The use of this equation enables one to express the product of any operator and the statistical matrix as the diagonal matrix element of the operator on the vector $|\Phi\rangle$.

Substituting the thermofield operators for the initial ones in the Hamiltonian $\tilde{H}_H + H$, we find the result to lowest order in n_q by omitting squared and higher terms in n_q:

$$\tilde{H}_H + H = H_0 + H_1 \tag{6a}$$

$$H_0 = \sum E(p\sigma) a^+_{p\sigma} a_{p\sigma} + \sum_q \omega(q) c^+_q c_q - \sum_q \omega(q) \tilde{c}^+_q \ \tilde{c}_q + \frac{(2S)^{1/2} A}{N^{1/2}}$$
$$\times \sum_{kp} (a^+_{k\mathrm{u}}\, c^+_{p-k} a_{p\mathrm{d}} + \mathrm{H.c.}) + \frac{A}{N^{1/2}} \sum_{kk_1q} (a^+_{k\mathrm{d}} a_{k_1\mathrm{d}} - a^+_{k\mathrm{u}} a_{k_1\mathrm{u}})\, c^+_q\, c_{k-k_1+q}$$
$$- \frac{A}{(2SN)^{1/2} N} \sum_{kk_1qq_1} a^+_{k_1\mathrm{u}} a_{k\mathrm{d}}\, c^+_{q_1}\, \tilde{c}^+_{k-k_1+q-q_1}\, c_q \tag{6b}$$

$$H_1 = \frac{A(2S)^{1/2}}{N^{1/2}} \sum_{kp} (a^+_{k\mathrm{u}} \sinh\theta(p-k)\, \tilde{c}_{p-k} a_{p\mathrm{d}} + \mathrm{HC})$$
$$+ \frac{A}{N} \sum_{kk_1k_2} (a^+_{k_1\mathrm{d}} \sinh\theta(k_1 - k)\, c^+_{k_2-k} \tilde{c}^+_{k_1-k} a_{k_2\mathrm{d}} + \mathrm{HC})$$
$$- \frac{A}{N} \sum_{kk_1k_2} (a^+_{k_1\mathrm{u}} \sinh\theta(k_1 - k)\, c^+_{k_2-k} \tilde{c}^+_{k_1-k} a_{k_2\mathrm{u}} + \mathrm{HC})$$
$$+ \frac{A}{(2SN)^{1/2}} \sum_{kk_1qq_1} a^+_{k\mathrm{d}} a_{k_1\mathrm{u}}\, \tilde{c}^+_q\, c^+_{q_1}\, c^+_{k-k_1+q-q_1} \sinh\theta(q). \tag{6c}$$

For brevity we omit the index β in the thermofield magnon operators $\tilde{c}$. We also neglect the difference between $\cosh\theta(q)$ and unity. This small quantity must be taken into account when calculating the effective mass renormalization, but it leads to higher order corrections when calculating the damping renormalization.

At $T = 0$, the non-vanishing part H_0 will be treated as the unperturbed Hamiltonian. It is made up of terms not involving creation or annihilation of thermofield holes. The perturbative part H_1 takes account of hole creation processes in the lowest order in temperature. This replaces the processes of magnon annihilation usually found in quantum mechanics. Since the Dyson–Maleev transformation is not unitary, it contributes non-Hermitian terms to the Hamiltonian (6) (e.g. there is no Hermitian conjugated counterpart to the last term in (6)). Nevertheless, one can see that this part must contain terms describing either the annihilation of two real magnons or the creation of two thermofield holes, and it is of higher order in temperature.

After transformation (5) we obtain the Hamiltonian in a simple form in which H_0 describes spin polaron states with infinite lifetime and H_1 unavoidably causes polaron decay because it creates a magnon hole with negative energy and a magnon with positive energy. This process substitutes for the usual electron–magnon scattering.

To calculate the damping and mobility of the spin polaron, we have to diagonalize H_0 and transform the free-electron operators $a^+_{p\sigma}$ and $a_{p\sigma}$ to polaron operators $\alpha^+_{p\sigma}$ and $\alpha_{p\sigma}$.

3. Canonical transformation to polaron operators

The transition to spin polaron operators can be made in several ways, for example by using the explicit form of the spin polaron wavefunctions [1–3]. We shall use the canonical transformation with the following unitary operator

$$\hat{U} = \mathrm{e}^{B} \qquad B = \sum_{kp} [(2S)^{1/2} b(k,p) a^{+}_{k\mathrm{d}} c_{k-p} a_{p\mathrm{u}} - (2S)^{1/2} b(k,p) a^{+}_{p\mathrm{u}} c^{+}_{k-p} a_{k\mathrm{d}}] \tag{7}$$

where $b(k,p)$ is a real function whose explicit form is defined below.

The canonical transformation (7) has been used in the theory of phonon polarons of intermediate coupling [19], but in those cases it does not lead to exact eigenfunctions. This is because the transformation to phonon polaron operators must include terms with any power of boson creation and annihilation operators. In the spin polaron case, these terms vanish because of spin conservation. The spin polaron in ferromagnets bears only a virtual boson, and the transformation (7) gives the precise diagonalization of the Hamiltonian H_0.

The transformed fermion and boson operators are

$$\alpha^{+}_{k\sigma} = \mathrm{e}^{B} a^{+}_{k\sigma}\, \mathrm{e}^{-B} \qquad \gamma^{+}_{k} = \mathrm{e}^{B} c^{+}_{k}\, \mathrm{e}^{-B}. \tag{8}$$

The function $b(k,p)$ is defined by the condition that the wavefunction $\alpha^{+}_{k-} |0\rangle$ would satisfy the Schrödinger equation with the Hamiltonian H_0. Here the state $|0\rangle$ corresponds to the fermion vacuum $|0\rangle_{\mathrm{el}}$ and thermofield vacuum $|\Phi\rangle$: $|0\rangle = |0\rangle_{\mathrm{el}} \otimes |\Phi\rangle$.

We only studied one-electron states, and the following identities are valid in the one-fermion subspace:

$$\alpha_{k\sigma}\alpha_{p\sigma_1} = 0 \qquad \alpha_{k\sigma}\alpha^{+}_{p-\sigma} = 0 \qquad \alpha_{k\sigma}\alpha^{+}_{p\sigma_1} = \delta_{kp}\delta_{\sigma\sigma_1}. \tag{9}$$

Expanding e^{-B} and summing the row taking into account identities (9), we obtain the expression

$$\begin{aligned}\mathrm{e}^{-B} = 1 &+ \sum_{kp} \alpha^{+}_{k\mathrm{u}}\alpha_{p\mathrm{u}}[(\cos\hat{\mathbf{L}}_1)_{kp} - \delta_{kp}] + \sum_{kp} \alpha^{+}_{k\mathrm{d}}\alpha_{p\mathrm{d}}[(\cos\hat{\mathbf{L}}_2)_{kp} - \delta_{kp}] \\ &+ \sum_{kpq} \alpha^{+}_{k\mathrm{u}}\alpha_{p\mathrm{d}}(2S)^{1/2} b(k,q) c^{+}_{k-q} (\sin\hat{\mathbf{L}}_2/\hat{\mathbf{L}}_2)_{qp} \\ &+ \sum_{kpq} \alpha^{+}_{k\mathrm{u}}\alpha_{p\mathrm{d}}(2S)^{1/2} b(k,q) c_{q-k} (\sin\hat{\mathbf{L}}_2/\hat{\mathbf{L}}_2)_{qp}\end{aligned} \tag{10}$$

where quasimatrices $\hat{\mathbf{L}}_2$ and $\hat{\mathbf{L}}_1$, with elements $L_2(q,p)$ and $L_1(q,p)$, are given by

$$L_1(q,p) = \left[2S \sum_{pqq_1} b(q,q_1) b(p,q_1)\, c^{+}_{q_1-q} c_{q_1-p}\right]^{1/2} \tag{11a}$$

$$L_2(q,p) = \left[2S \sum_{pqq_1} b(q,q_1) b(p,q_1)\, c_{q_1-q} c^{+}_{q_1-p}\right]^{1/2}. \tag{11b}$$

The matrix functions in (10) must be treated as the sum of corresponding series. These series only contain terms with even powers of matrices $\hat{\mathbf{L}}_1$ and $\hat{\mathbf{L}}_2$, so there are no terms with fractional powers of operators in (10).

From the definition (11), taking into account (9) and substituting exp(B) from (10) for (8), we obtained the following fermion operator transformation laws:

$$\alpha^+_{kd} = \sum_q a^+_{qd}(\cos \hat{L}_1)_{kq} + \sum_{pq} a^+_{pu}(2S)^{1/2} b(p,q) c^+_{p-q} \left(\frac{\sin \hat{L}_2}{\hat{L}_2}\right)_{qk} \tag{12a}$$

$$\alpha^+_{ku} = \sum_q a^+_{qu}(\cos \hat{L}_2)_{kq} + \sum_{pq} a^+_{pd}(2S)^{1/2} b(p,q) c_{q-p} \left(\frac{\sin \hat{L}_1}{\hat{L}_1}\right)_{qk}. \tag{12b}$$

From the definitions of $\hat{L}_1$ and $\hat{L}_2$ we can derive commutation relations:

$$\sum_q b(k,q) c^+_{k-q} \left(\frac{\sin \hat{L}_2}{\hat{L}_2}\right)_{qp} = \sum_q \left(\frac{\sin \hat{L}_1}{\hat{L}_1}\right)_{kq} b(p,q) c^+_{p-q} \tag{13a}$$

$$\sum_q b(k,q) c_{q-k} \left(\frac{\sin \hat{L}_1}{\hat{L}_1}\right)_{qp} = \sum_q \left(\frac{\sin \hat{L}_2}{\hat{L}_2}\right)_{kq} b(p,q) c_{q-p}. \tag{13b}$$

Using these relations we can easily prove that the transformation (10) is unitary.

The polaron wavefunction $\alpha^+_{kd}|0\rangle$ must be defined from expression (12). To do so, we must calculate only the vacuum matrix elements. Further formulae look much simpler if we take into account the relations $c_q c^+_k|0\rangle = 0$ (where $q \neq k$) and $c_q|0\rangle = 0$. As a result, only the diagonal elements of these operators do not vanish, and the wavefunction reduces to

$$\alpha^+_{kd}|0\rangle = a^+_{kd} \cos G(k)|0\rangle + \sum_q b(q,k) a^+_{qu}(2S)^{1/2} c^+_{k-q} \frac{\sin G(k)}{G(k)}|0\rangle. \tag{14}$$

Similarly,

$$\alpha^+_{ku}|0\rangle = a^+_{ku}|0\rangle. \tag{15}$$

In (14),

$$G(k) = 2S \sum_q b^2(k,q) = \langle 0|L^2_2(k,k)|0\rangle.$$

Note that a formula similar to (14) for a conventional polaron was obtained in [20].

The function $b(k,q)$ has not been determined yet. We define it using the condition that (14) is the eigenfunction of Hamiltonian H_0 (i.e. the terms linear in γ^+ in transformed H_0 must vanish). This is sufficient since the terms $\gamma^+\gamma^+$ must also vanish due to spin conservation, and the normal terms $\gamma^+\gamma$ turn out to be zero in the vacuum state.

On substituting the wavefunction (14) into the Schrödinger equation, we obtain

$$\hat{H}_0 \alpha^+_{kd}|0\rangle = \mathscr{E}(k)\alpha^+_{kd}|0\rangle \tag{16a}$$

where

$$\mathscr{E}(k) = E(kd) + \frac{2AS \tan G(k)}{N^{1/2} G(k)} \sum_q b(k,q) \tag{16b}$$

with the function $b(k,q)$ defined by

$$b(k,q) = G(q)[E(qu) - \mathscr{E}(q)]/2S(N)^{1/2} \tan G(q)[\mathscr{E}(q) - E(ku) - \omega(q-k)]. \tag{17}$$

Excluding $b(k,q)$ from these formulae, we find the equation for the spin polaron spectrum $\mathscr{E}(k)$ in the standard form [2]:

$$1 - \frac{2AS}{2AS + \mathscr{E}(k) - E(kd)} = \frac{A}{N} \sum_q \frac{1}{\mathscr{E}(k) - E(qu) - \omega(k-q)}. \tag{18}$$

In the case of narrow bands $AS \gg \Delta E$ (ΔE is the band width), the function $b(k, q) = -1/2S(N)^{1/2}$, and transformation (8) coincides with that found in [12], based on this extreme case.

When the one-electron spin-down Green function for zero temperature is defined in the usual way,

$$\mathscr{G}(\omega, p) = \mathrm{i}\, T \int_{-\infty}^{\infty} \mathrm{e}^{\mathrm{i}\omega t - \varepsilon|t|} \langle 0|\, \mathrm{e}^{\mathrm{i}H_0 t} a_{p\mathrm{d}}\, \mathrm{e}^{-\mathrm{i}H_0 t} a^+_{p\mathrm{d}} \,|0\rangle \,\mathrm{d}t \tag{19}$$

it can be reduced to

$$\mathscr{G}(\omega, p) = z(p)/(\omega - \mathscr{E}(p) + \mathrm{i}\varepsilon) \tag{20}$$

near its pole, where $z(p) = \cos^2 G(p)$ is the polaron wavefunction renormalization constant.

4. Magnetic polaron damping and mobility

To calculate the mobility, we shall use the results of Langreth and Kadanoff [21], who dealt with a similar polaron problem in ionic crystals. They obtained a perturbation expansion of the mobility as a power series in the coupling constant and electron density. They found that the current–current correlator in the first non-vanishing order is reduced to the product of two one-particle Green functions. For the mobility, they obtained the formula

$$\mu = \frac{e\hbar^3}{6k_\mathrm{B} T n_\mathrm{e} m^2} \int p^2 \frac{\mathrm{d}^3 p}{(2\pi)^3} \int \frac{\mathrm{d}\omega}{2\pi} [A(p, \omega)]^2 \exp\left(-\frac{\omega - \zeta}{k_\mathrm{B} T}\right) \tag{21}$$

where n_e is the electron density, ζ is the chemical potential, $\zeta/k_\mathrm{B}T \to -\infty$, p is the wavevector,

$$A(p, \omega) = 2z(p)[\tau(p)/\hbar]^{-1}/\{[\omega - \mathscr{E}(p)]^2 + [\tau(p)/\hbar]^{-2}\} \tag{22}$$

is the spectral weight function for the one-electron Green function, $\tau^{-1}(p) = 2\Gamma(p, \mathscr{E}(p))/\hbar$ is the reciprocal polaron lifetime and $\Gamma(p)$ is the polaron damping determined in the usual way as $\Gamma(p) = \mathrm{Im}\, \Sigma(p, \mathscr{E})$ where Σ is the self-energy.

To calculate the mobility to the first non-trivial order, we can take the values of $\mathscr{E}(p)$ and $z(p)$ from Green's function (20), but the finite value of $\tau(p)$ should be obtained from the Hamiltonian H_1. To the second order in H_1, we have

$$\Sigma(p, \omega) = \sum_s \frac{z(s)}{\omega - E(s) + \mathrm{i}\varepsilon} |\langle p|H_1|s\rangle|^2 \tag{23}$$

where s denotes eigenstates of H_0 with energy $E(s)$ and $|p\rangle = \alpha^+_{p\mathrm{d}}|0\rangle$ is the polaron wavefunction. Taking the imaginary part, we obtain

$$\Gamma(p) = \pi \sum_s z(s) |\langle s|H_1 \alpha^+_{p\mathrm{d}}|0\rangle^2 \delta(E(s) - \mathscr{E}(p)). \tag{24}$$

We studied polaron states with small wavevectors p and low energies, thus $|s\rangle$ in (23) must contain only spin-down states. The spin-up states have energies about $2AS$ or higher so they cannot satisfy energy conservation criteria. The boson states in $|s\rangle$ include one thermofield magnon hole and one real thermofield magnon. This is a result of the two magnon interaction in H_1. To obtain the value of damping, from (23), we calculate the function $H_1\alpha^+_{pd}|0\rangle$. Instead of transforming the operator H_1 using transformation (8), it is convenient to calculate this function in terms of the initial operators a and c. If we retain only the terms linear in n_q from (7) and (14), we find

$$H_1\alpha^+_{pd}|0\rangle = \frac{A}{N}\sum_{qq_1} c^+_q c^+_{q_1} \sinh\theta(q)\cos G(p)\frac{E(p)-E(p-q)-\omega(q_1)}{\mathscr{E}(p)-E(p-q_1\mathrm{u})-\omega(q_1)}$$

$$\times\left(a^+_{p+q-q_1\mathrm{d}} - \frac{1}{(2S)^{1/2}N}\sum_{p_1} a^+_{p_1+q+q_1\mathrm{u}}\tilde{c}^+_{p-p_1}\right)|0\rangle. \tag{25}$$

The states $|s\rangle$ have wavefunctions of the type

$$|s\rangle = \alpha^+_{p_1\mathrm{d}}\gamma^+_q\tilde{\gamma}^+_{q_1}|0\rangle = \mathrm{e}^B a^+_{p_1\mathrm{d}}c^+_q\tilde{c}^+_{q_1}|0\rangle \tag{26}$$

and $z(s) = z(p_1)$. As we calculate the matrix element in terms of the initial operators, we use the latter equality for $|s\rangle$ in (26). To simplify the calculations, note that $\tilde{c}^+_{q_1}$ commutates with e^B. Although the commutator $[\mathrm{e}^B, \tilde{c}^+_q]$ does not equal zero, it can nevertheless be neglected because its contribution to the lifetime is of the order $1/N$ (the single electron is not able to change real magnon states). So we have the convenient formulae

$$|s\rangle = \tilde{c}^+_{q_1}c^+_q\,\mathrm{e}^B a^+_{p_1\mathrm{d}}|0\rangle = \tilde{c}^+_{q_1}c^+_q\,\alpha^+_{p_1\mathrm{d}}|0\rangle \tag{27a}$$

$$H_0\tilde{c}^+_{q_1}c^+_q\,\alpha^+_{p\mathrm{d}}|0\rangle = (\omega_q - \omega_{q_1} + \mathscr{E}(p))\tilde{c}^+_{q_1}c^+_q\,\alpha^+_{p\mathrm{d}}|0\rangle. \tag{27b}$$

Substituting (27) and (25) in (24) we obtain, after some tedious calculations,

$$\Gamma(k) = \frac{\pi}{N^2}\sum_{qq_1}\delta(\mathscr{E}(k+q-q_1)+\omega(q_1)-\omega(q)-\mathscr{E}(k))$$

$$\times\left(\frac{E(k)-E(k-q_1)-\omega(q_1)}{\mathscr{E}(k)-E(k-q_1)-2AS-\omega(q_1)}\right)^2 R^2(k,k+q-q_1)n_q \tag{28a}$$

$$R(k) = (1/2S)\cos G(k)\cos^2 G(k+q-q_1)[E(k+q-q_1)+2AS-\mathscr{E}(k+q-q_1)]. \tag{28b}$$

Let us compare this formula with those already known. In the limit $\omega_q \to 0$, the relations $\mathscr{E}(k) = \mathscr{E}(k+q-q_1)$ and $E(k) = E(k+q-q_1)$ are valid and formula (28) coincides with the result of [16] to the lowest order in n.

In the extreme case of narrow bands or a high value of atomic spins, $2AS \gg 1$, the value $R(k,p) = A$ and (28) coincides with the results of [11, 12].

For small wavevectors we can calculate the polaron damping explicitly. Polaron and magnon spectra can be represented in the usual way as $E(k) = \hbar^2k^2/2m$, $\mathscr{E}(k) = \mathscr{E}(0) + \hbar^2k^2/2m^*$, $\omega(q) = \omega_0 + \hbar^2q^2/2M$, where m is the free-electron mass; m^* is the polaron effective mass, M is the magnon effective mass and $\mathscr{E}(0)$ is the shift of the bottom band caused by polaron formation. Further, we can imply the validity of the inequalities $M \gg m$, $M \gg m^*$, $AS \gg \omega_0$, $AS \gg k_{\mathrm{B}}T$. Since polaron thermal wavevectors are much smaller than magnon thermal wavevectors, we have, from (28),

$$\Gamma(k) = \frac{v^2m^*kR^2}{4\pi^2\hbar^2}\int p^2\mathrm{d}p\left(\frac{p^2+\kappa^2}{p^2+\lambda^2}\right)^2 n_p \tag{29}$$

with $\kappa^2 = 2\omega_0 m/\hbar^2$, $\lambda^2 = (2AS-\mathscr{E}(0))2m/\hbar^2$, where v is the unit cell volume, $R =$

$R(0,0) = A\cos^3 G(0)\,(1 - \mathscr{E}(0)/2AS)$. In the case of intermediate or strong s–f coupling, λ is not small. At low temperatures, we use the inequality $p \ll \lambda$, and omit p^2 from the denominator in (29).

For the temperature range $k_B T \gg \omega_0$, when we use the identity $\hbar^2/2M = ISa^2$ (where $a = v^{1/3}$ and I is the Heisenberg exchange integral), we obtain for the polaron reciprocal lifetime the formula

$$\tau^{-1}(k) = 2\Gamma(k)/\hbar = [\hbar k \cos^6 G(0)/8maS^2](k_B T/IS)^{7/2}\,\Phi_0(m^*/m) \qquad (30a)$$

in which $\Phi_0 = 15\,\pi^{1/2}\,\zeta(7/2)/64\,\pi^3 = 1.5 \times 10^{-2}$, where ζ is the Riemann function.

So far, we have treated the spin polaron almost as a free particle, which requires that it fulfil the condition $\Gamma \ll \mathscr{E}(k)$, that is

$$ka \gg [\cos^6 G(0)/4S^2](k_B T/IS)^{7/2}\,\Phi_0. \qquad (30b)$$

For both extreme cases of broad and narrow band widths with $2S \gg 1$, $\cos G(0) = 1$.

The result given by (30) is well known, but at temperatures $k_B T \ll \omega_0$, for which magnon energies must be taken into account explicitly, the expression for polaron lifetime has a different form. Retaining κ and dropping p^2 from the nominator of (29), we find

$$\tau^{-1}(k) = [vm^* k\omega_0^2 \cos^6 G(0)/32\,\hbar^3 \pi^{3/2} S^2](k_B T/IS)^{3/2}\exp(-\omega_0/k_B T). \qquad (31)$$

Finally, integrating (21) over polaron wavevectors, and taking into account the fact that the integrand is not small only for low energies, we obtain for the mobility the formula

$$\mu = \frac{e\hbar^3}{12k_B T n_e m^2}\int \frac{p^2\,\mathrm{d}^3p\,z^2}{(2\pi)^3\Gamma(p)}\exp\left(-\frac{\mathscr{E}(p)-\zeta}{kT}\right)$$
$$= [z^2\hbar e(2m^*)^{1/2}/3m^2(\pi k_B T)^{1/2}](\partial\tau^{-1}(k)/\partial k)^{-1}. \qquad (32)$$

As we can see from this final formula, increasing temperature causes the mobility to decrease extremely rapidly.

In conclusion, we note that magnetic field has a great effect on ω_0, and formulae (31) and (32) successfully describe the negative magnetoresistance of ferromagnetic semiconductors.

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